Random Infinite Tree and Dependent Poisson Diffusion Process for Nonparametric Bayesian Modeling in Multiple Object Tracking

Bahman Moraffah and Antonia Papandreou-Suppappola

School of Electrical, Computer, and Energy Engineering, Arizona State University, Tempe, AZ

Main Objective

Problem Statement

- Track multiple objects with time-varying (TV) cardinality and unknown measurement to object association
- Challenge: robustly associate objects at current time step with objects tracked at previous time step

Goals

- Jointly estimate TV object label and cardinality
- Capture time-dependency among multiple object states
- Simple to implement algorithm with high estimation accuracy

Paper Contributions

- Dependent Poisson diffusion process as prior on object state
  - Nonparametric distribution over time-evolving trees
  - Can model hierarchies, similar to Dirichlet diffusion tree; also captures TV dependencies of object states
  - Provides joint estimation of TV object state and cardinality; state estimated by selecting path connected to each leaf, and object labels are inferred by tracing random tree path
- Dependent mixture model updates object cardinality and posterior distribution; inference using MCMC sampling
- Achieve higher estimation accuracy and lower computational cost at lower SNR values
- Dependent Poisson diffusion tree attains frequentist minimax rate of convergence

Related Approaches

Random Finite Set (RFS)Theory

- RFS represent uncertainty in the number and state of objects, multiple-object filtering (Mahler, 2007)
- e.g., Labeled multi-Bernoulli filtering (LMB) (Reuter, Vo & Vo, 2014)

Evolutionary Clustering

- Does not capture objects enter/leave scene [Chakrabarti 2006]
- Assume known number of clusters, no time-dependency model

Bayesian Nonparametric Models

- Dependent Dirichlet process (DDP) as prior [authors, Asilomar 2018]
- Hierarchical Dirichlet process: correlated modes [Fox 2011]
- Dynamic clustering via DDP [Campbell 2013]
- Bayesian inference for linear dynamic model [Caron 2007]

Dependent Poisson Diffusion Process (D-PoDP) & random Tress

- Modeling uncertainty over trees; path/branch generated by diffusion process (generate samples using Brownian motion at $k = 0$)
- Branching probability: probability of selecting a branch vs diverging, depends on number of samples previously followed same branch
- Dependent as prior can incorporate time-dependent learned information
- Problem: transition kernel $p_{\theta_k}(x_k|\, x_{k-1})$ with unknown parameter θ_k
  - Use a dependent diffusion process on a tree as prior on θ_k
  - Tree leaf/node: object state, branch: cluster of states in a hierarchy
  - Find trajectory of each object by tracing path on tree
  - Predict and update number of objects at each time

Proposed D-PoDP Algorithm

- At time k, N_k objects with state x_k enter, leave or remain in scene
- Transition $(k-1)$ to k: object leave branch with probability $(1 - P_{k|k-1})$ or survive with probability $P_{k|k-1}$ and its state transitions with distribution $p_{\theta_k}(x_k|\, x_{k-1})$ with unknown parameter θ_k
- Assign probability to survived branch a

$$p_a \propto |S_{a,k-1}| + |S_{a,k|k-1}| - \gamma$$

where $|S_{a,k-1}|$ is the number of objects with common branch a

- For new object, assign probability to new branch δ, $p_\delta \propto \zeta - |V_{B,k|k-1}|\gamma$ where $|V_{B,k|k-1}|$ is the number of survived branch nodes
- At time k, draw $\tilde{N}_{\ell,k|k-1}$ objects from a Poisson process

$$\tilde{N}_{\ell,k|k-1} \sim \text{Po}\left(\frac{\lambda p_a}{2|S_{a,k|k-1}|}\right) \quad \text{for all} \quad \theta_{\ell,k|k-1} \in S_{a,k|k-1}$$

- Generate $\tilde{N}_{\ell,k|k-1}$ by diffusion process given $\theta_{\ell,k|k-1}$, transition to time k
- Draw $\tilde{N}_{\delta,k|k-1}$ from a Poisson with parameter $\lambda\, p_\delta/2$ and generate $\tilde{N}_{\delta,k|k-1}$ points from the base distribution of θ_0
- Draw $x_{\ell,k} \mid \theta_{\ell,k} \sim G(\cdot \mid \theta_{\ell,k})$, for distribution G and $\tilde{N}_k = \sum_\ell \tilde{N}_{\ell,k|k-1}$

Dependent Mixture Model and Inference

Use constructed prior as mixing distribution to infer measurement distributions

- Select parameter $\theta_{\ell,k}$ at time k with probability π_ℓ proportional to $n_{\ell k}$, number of measurements that already selected same parameter and number of object with the shared branch

$$\pi_\ell \propto n_{\ell,k} + |S_{a,k-1}| \quad for \;\; \theta_{\ell,k-1} \in S_{a,k-1}, \theta_{\ell,k} \in \tilde{V}_k$$

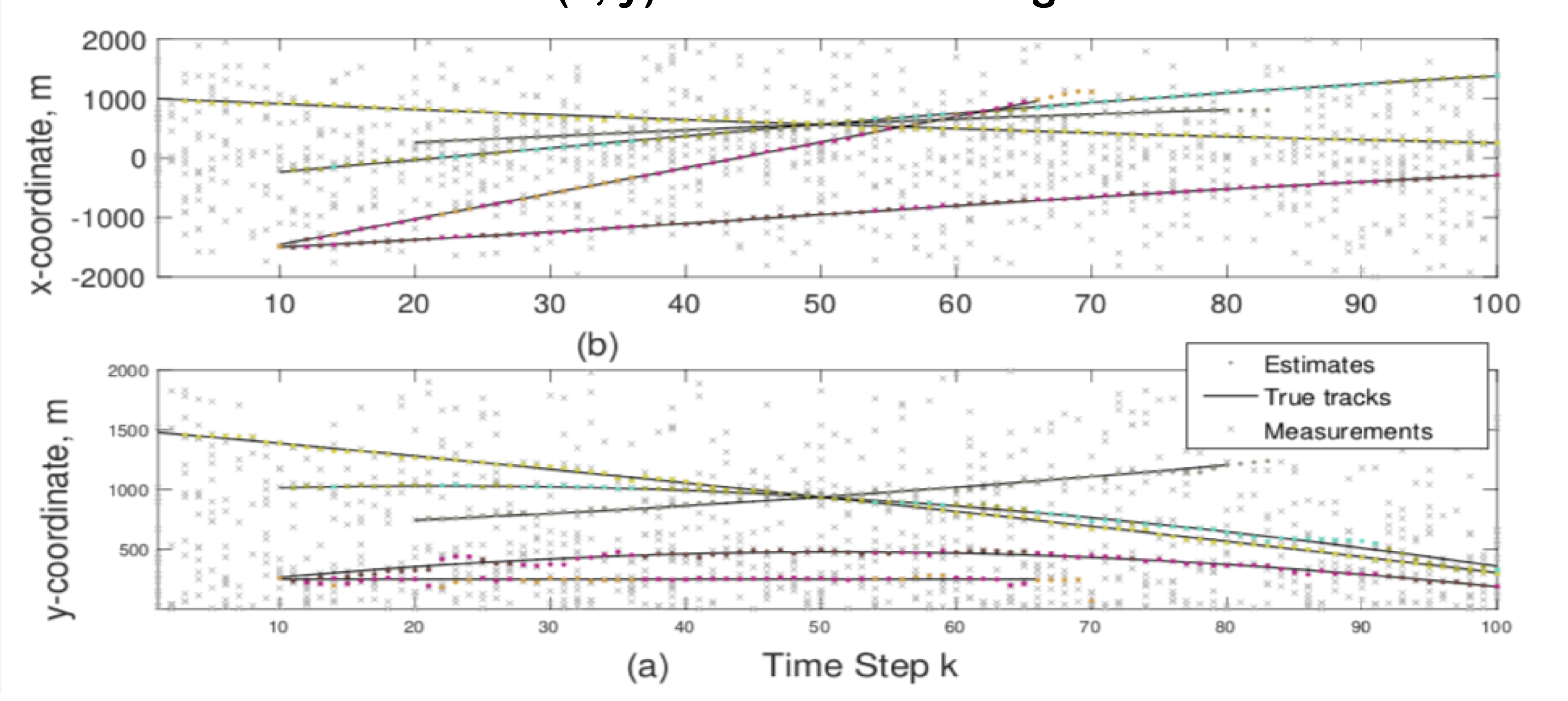
- New parameters generated with probability proportional to ξ

Dependent mixture model $\mathbf{z}_{\ell,k} \mid \mathbf{x}_{\ell,k}, \theta_{\ell,k}, \pi_{\ell,k} \sim F(\mathbf{x}_{\ell,k}, \theta_{\ell,k})$

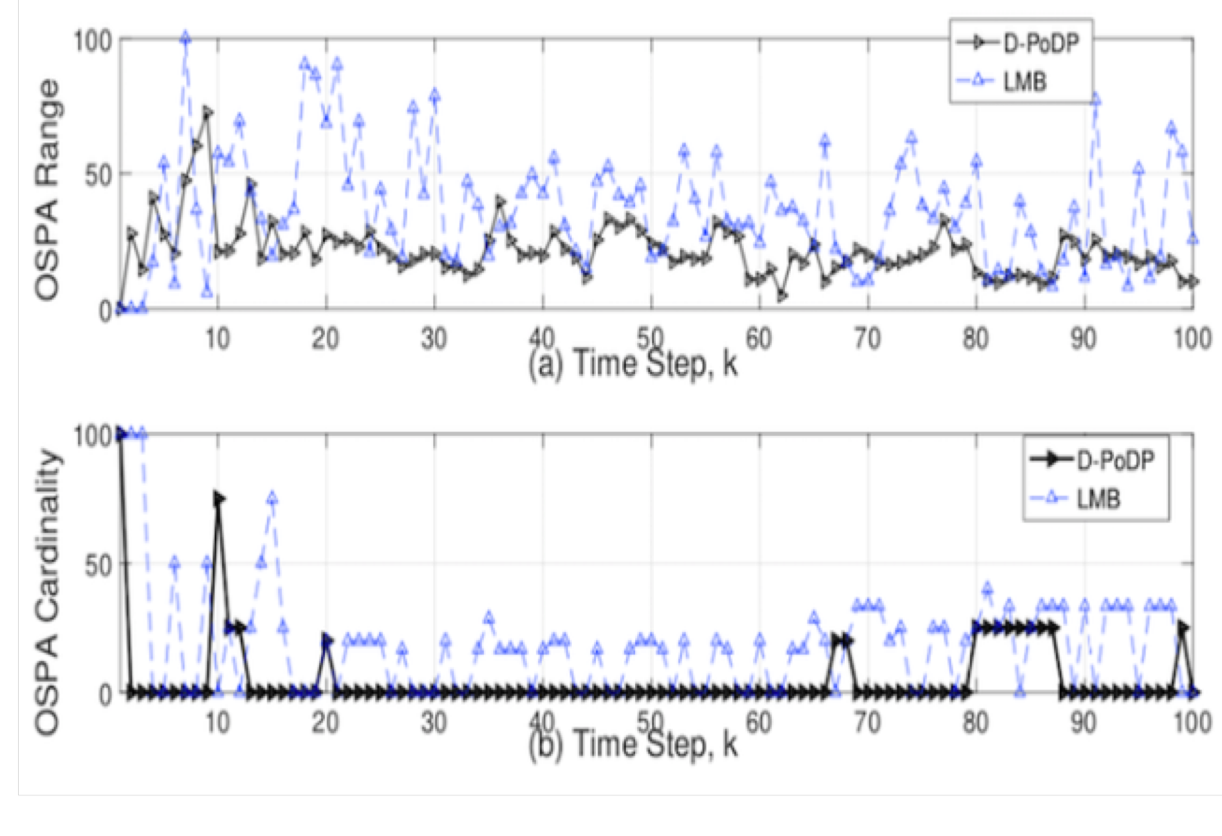
Use MCMC sampler for inference and find posterior distribution

Simulation Results: maximum 5 objects with TV cardinality

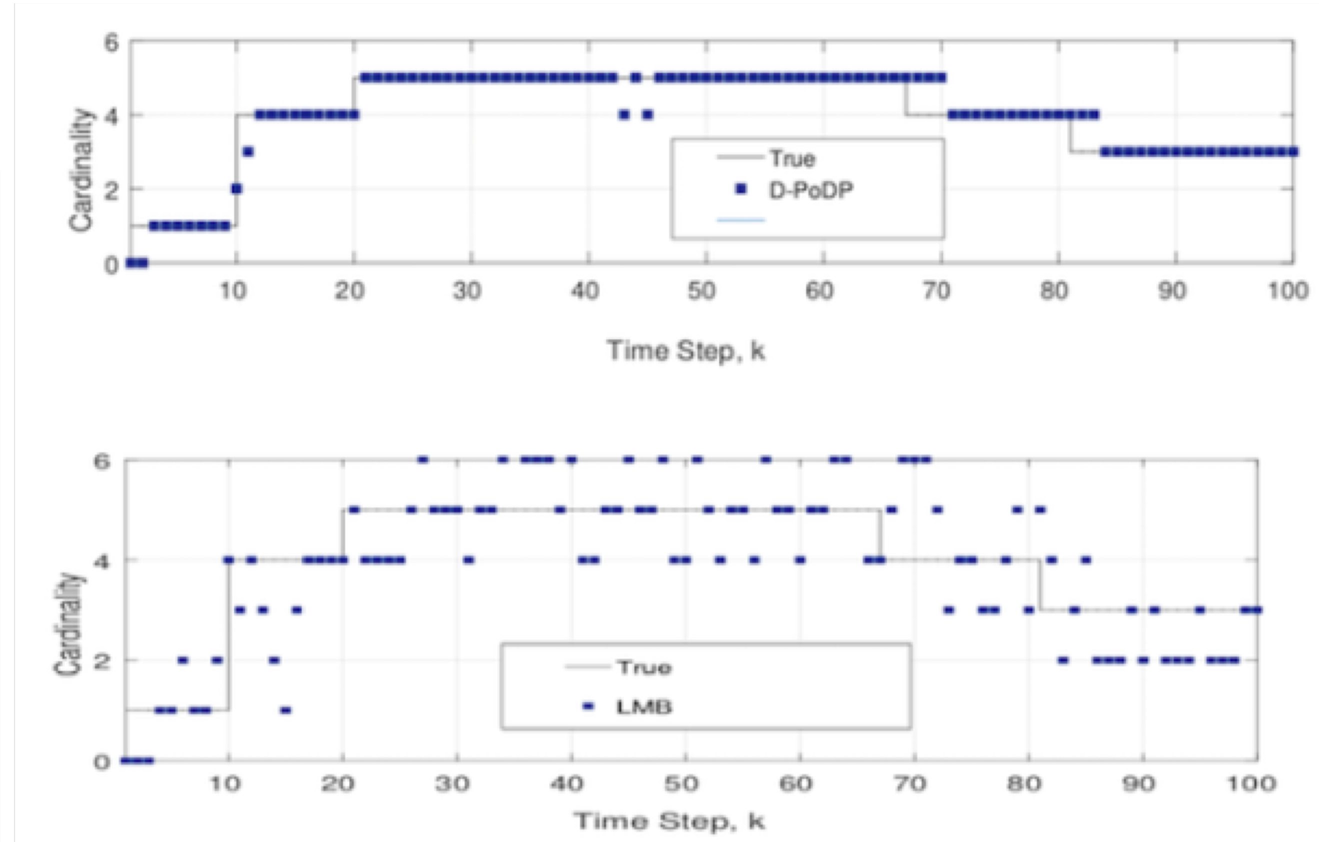
Actual and estimated (x, y) coordinates using newD-PoDP method



OSPA: new D-PoDP and labeled multi-Bernoulli (LMB) filtering



Learned cardinalities: new D-PoDP and LMB



Paper: https://ieeexplore.ieee.org/document/8682370