Nonparametric Bayesian Methods and the Dependent Pitman-Yor Process for Modeling Evolution in Multiple Object Tracking

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Challenges of dynamic multiple object tracking problem

- **Track unknown time-varying number of objects (cardinality)**
- **Objects leave, enter or stay in scene: unknown state label/identity**
- **Multiple observations from sensing modalities, possibly dependent, and unknown measurement-to-object associations**
- **Multiple environmental conditions: high noise levels, clutter, interference**

previous time step *(k-1)*

time step *k*

Multiple object tracking methods

Bayesian estimation of object state posterior using

- **physics-based models** **(e.g., constant velocity, maneuvering)**
- **parametric models** **(e.g., multi-Bernoulli)**

- **Joint probabilistic data association, multiple hypothesis testing, joint multitarget probability density, probability hypothesis density** (PHD)
- **Labeled multi-target multi-Bernoulli**: multi-Bernoulli RFS approximate multi-object posterior; labeled RFS estimate target identity, requires high probability of detection **[Vo & Vo 2012, 2014]**

Bayesian nonparametric modeling

Model underlying structure/distribution to learn and make predictions from data

- **Parametric model: family of distributions with finite number of parameters**
- **Nonparametric model: distributions with infinite dimensional parameter space (parameters can grow with complexity of observation)**

e.g., Gaussian mixture model (GMM) requires fixed number of clusters vs Dirichlet process (DP) mixture adapts number of clusters based on data

Nonparametric prior on object states for tracking

- Hierarchical DP: prior on unknown number of modes [E. Fox 2009]
- Bayesian inference: DP mixtures for noise in dynamic system [F. Caron 2008]
- Dependent DP prior, random finite tree: time-varying cardinality & label [B. Moraffah 2018, 2019]

Problem Formulation

- **Unknown state vector of ℓ th object:** $\mathbf{x}_{\ell,k}, \quad \ell = 1, \ldots, N_k$

object cardinality

- **If object is present at time $(k-1)$ and k:**

$$\mathbf{x}_{\ell,k} = f_k(\mathbf{x}_{\ell,k-1}) + \mathbf{u}_{\ell,k-1}$$

transition function **modeling error**

- **Measurement vector:** $\mathbf{z}_{m,k}, \quad m = 1, \ldots, M_k$

Assumptions: each measurement generated by only one object & measurements are independent of one another

- **If mth measurement originated from ℓ th object**

$$\mathbf{z}_{m,k} = h_k(\mathbf{x}_{\ell,k}) + \mathbf{w}_k$$

relation between measurement & state **measurement noise**

Multiple object tracking using dependent Pitman-Yor process

- Object state label and cardinality at current time step depends on:
  - labeled states at previous time step
  - previously labeled objects at current time step
- Dependent Pitman-Yor (DPY) process **to incorporate learning algorithm as prior over time-evolving object state distribution based on measurements**
  - Fully capture state dependence as it models collections of random distributions related but not identical; process realizations are dependent
  - Accurately estimate time evolving object trajectory and cardinality
  - Compared to dependent Dirichlet process (DDP): more available clusters to capture full dependency & likely to have less popular clusters
    - DDP: expected number of clusters $\alpha \log(N)$
    - DPY: expected number of clusters αN^d

# of objects

concentration

discount

Construction of multiple state prior distribution

Cluster label assignments with unknown cluster parameter θ_k

Three possible scenarios for an object in scene at time step k

Scenario 1:

Object ℓ placed in survived & transitioned cluster from time $(k-1)$ occupied by previous $(\ell-1)$ clustered objects at time k with probability

$$\Pi_1\Big(\text{Choose } j\text{th cluster}|\theta_{1,k},\ldots,\theta_{\ell-1,k}\Big) \propto [V^*_{k|k-1}]_j + [V_k]_j - d$$

normalization constant: $$\sum_i [V^*_{k|k-1}]_i + \sum_i [V_k]_i + \alpha$$

Construction of multiple state prior distribution

Scenario 2:

Object ℓ placed in survived & transitioned cluster from time $(k-1)$ not uet occupied by previous $(\ell-1)$ clustered objects at time k with probability

$$\Pi_2\Big(\text{Choose } j\text{th cluster not yet selected}|\theta_{1,k},\ldots,\theta_{\ell-1,k}\Big) \propto [V^*_{k|k-1}]_j - d$$

jth cluster size after transitioning

discount parameter

Scenario 3:

Object ℓ not placed in existing cluster from time $(k-1)$, generate new cluster at time k with probability

$$\Pi_3\big(\text{Create new cluster }|\theta_{1,k},\ldots,\theta_{\ell-1,k}\big) \propto |V_k|\, d + \alpha$$

Number of created clusters

concentration parameter

Bayesian Inference

- **Given configurations at time $(k-1)$, conditional distribution is Pitman-Yor**

$$\text{DPY}_k \mid \text{DPY}_{k-1} \sim \mathcal{PY}\Big(d, \alpha, \{\Pi_1, \Pi_2, \Pi_3\}\Big)$$

- **Prior state distribution**

$$p\big(\mathrm{x}_{\ell,k} \big| \mathrm{x}_{1,k}, \ldots, \mathrm{x}_{\ell-1,k}, \mathrm{X}_{k|k-1}, \Theta^*_{k|k-1}, \Theta_k\big) = \begin{cases} \mathbb{Q}_{\underline{\theta}}\big(\mathrm{x}_{\ell,k-1}, \mathrm{x}_{\ell,k}\big) f\big(\mathrm{x}_{\ell,k} \big| \theta^\star_{\ell,k}\big) \\ \mathbb{Q}_{\underline{\theta}}\big(\mathrm{x}_{\ell,k-1}, \mathrm{x}_{\ell,k}\big) \zeta\big(\theta^\star_{\ell,k-1}, \theta^\star_{\ell,k}\big) f\big(\mathrm{x}_{\ell,k} \big| \theta^\star_{\ell}(k)\big) \\ \int_\theta f\big(\mathrm{x}_{\ell,k} \big| \theta\big) dH\big(\theta\big) \end{cases}$$

Transition probability kernel

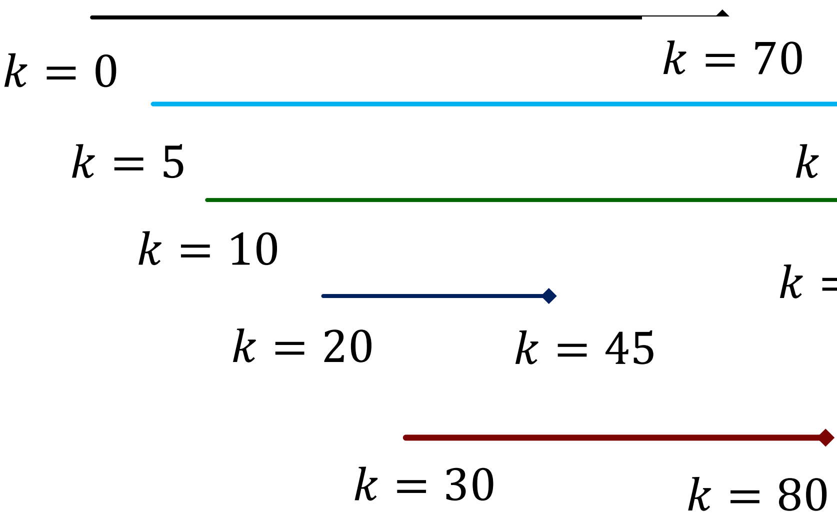
base distribution

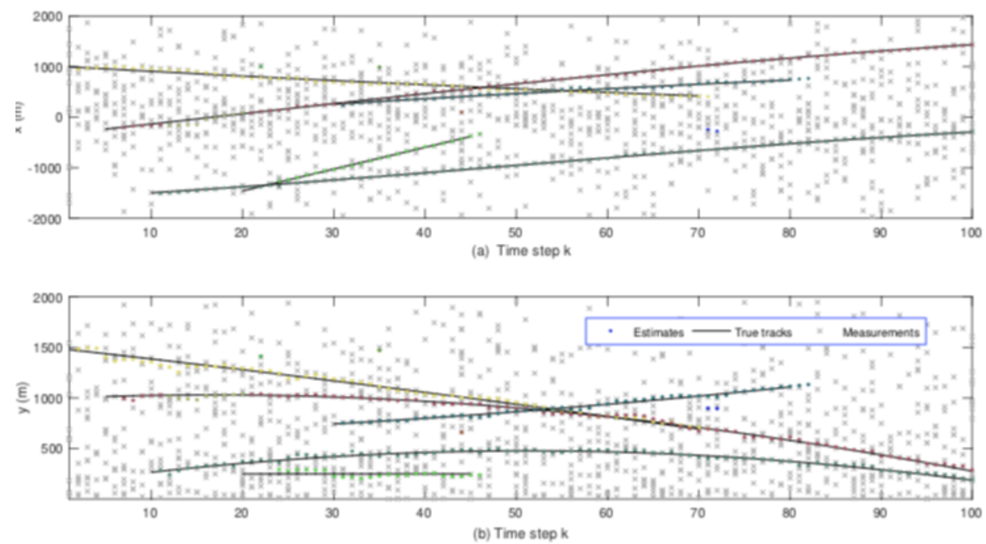
Transition kernel for parameters

$f(\cdot|\theta)$ **is derived from physical-based model**

- **Dirichlet process mixture to learn measurement-to-object associations**
- **Using measurements and physical model, compute posterior distribution** $\mathbf{x}_{\ell,k} | \mathbf{z}_{l,k}, \theta^\star_{\ell,k}$ **using an MCMC method (Gibbs sampling)**

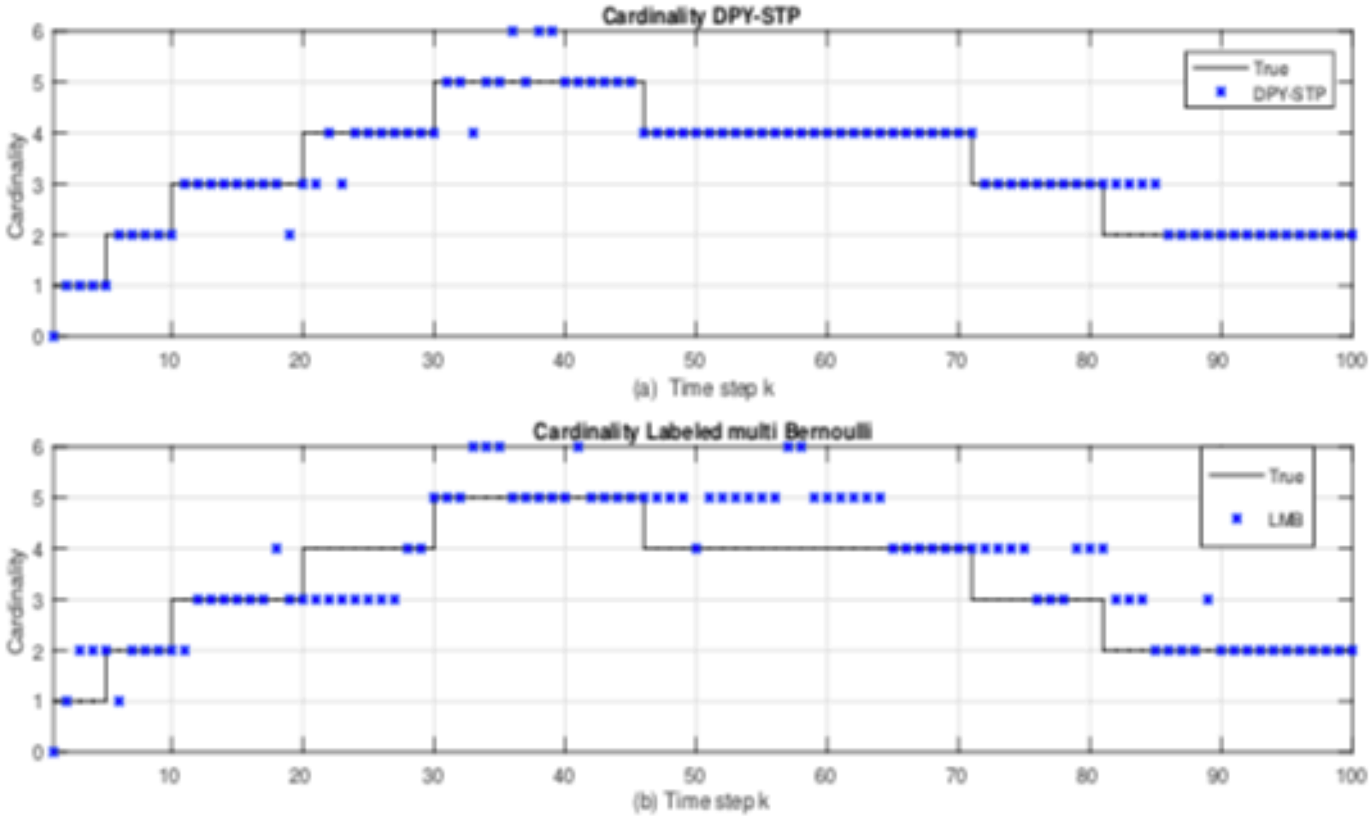
Time step of object entering and leaving 2-D scene





True and DPY estimated (x,y) coordinates

Simulation: tracking 5 objects, DPY vs labeled multi-Bernoulli (LMB)

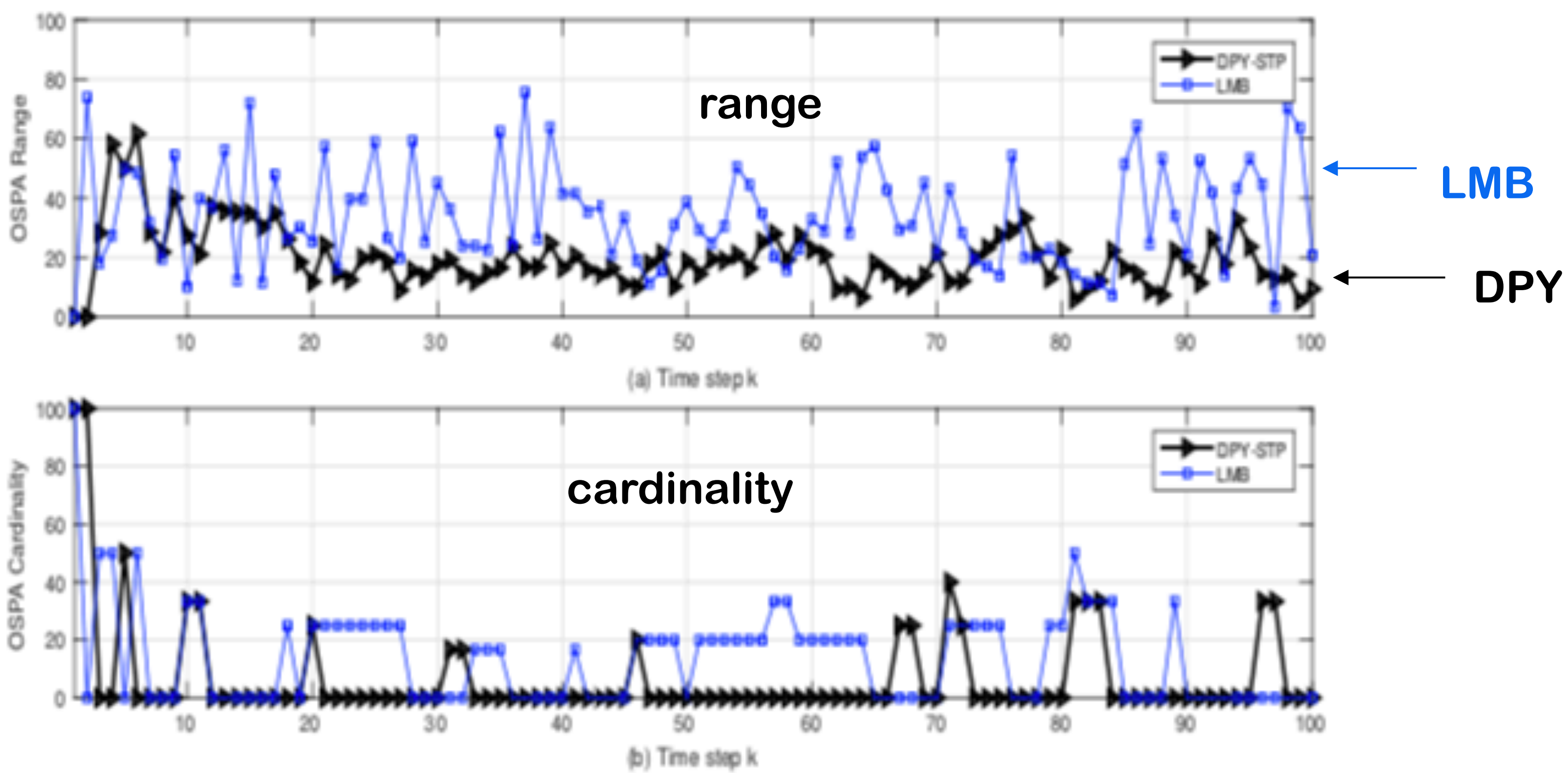


cardinality estimated with DPY

cardinality estimated with LMB

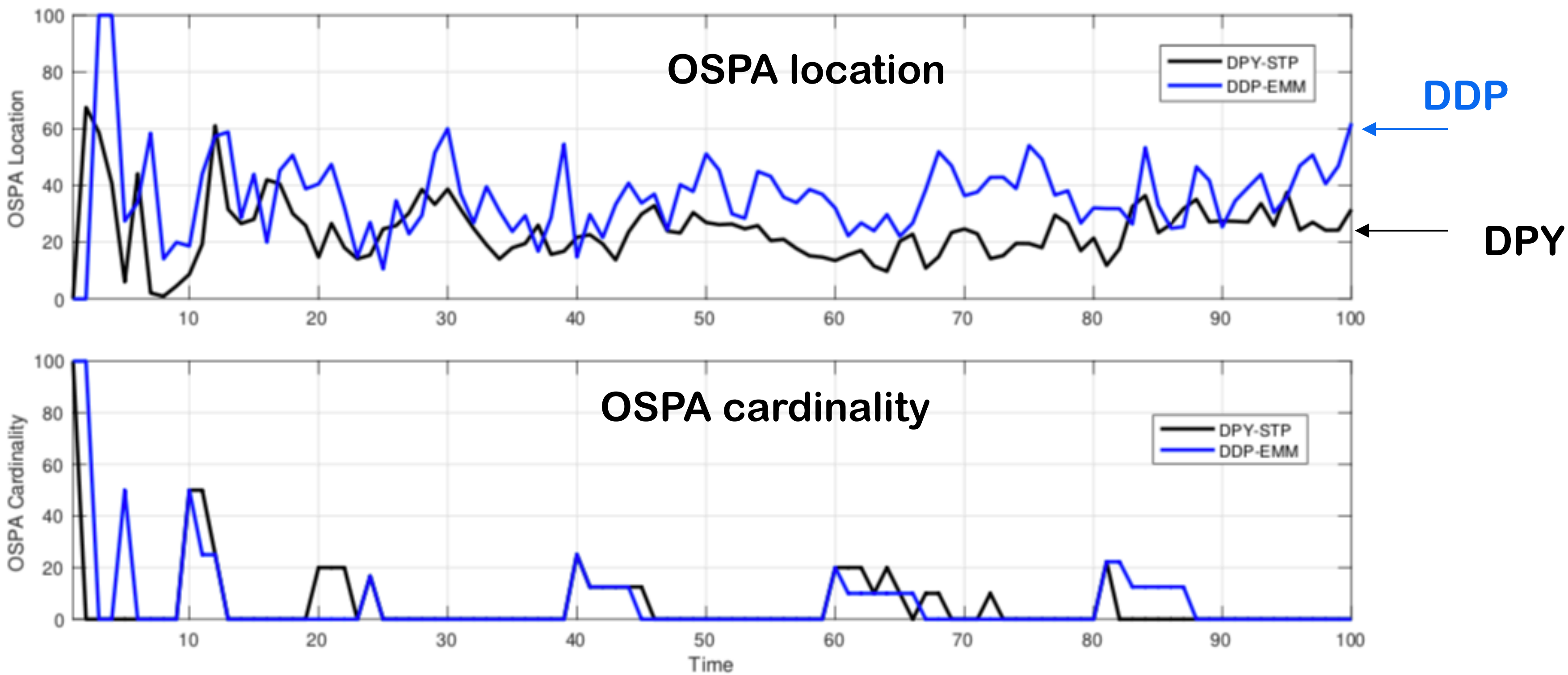
Simulation: tracking 5 objects, DPY vs labeled multi-Bernoulli (LMB)

OSPA (optimal sub-pattern assignment metric) comparison



Simulations: Comparison of dependent process use

OSPA comparison:
depend Dirichlet process (DDP) & dependent Pitman-Yor process (DPY)



Conclusions

- **Novel family of nonparametric processes naturally capture computational and inferential needs of a multiple object tracking problem**
- **Exploit dependent Pitman-Yor process to model dependencies in state prior**
- **Use Dirichlet process mixture to learn associations between measurements and objects**
- **Overall nonparametric Bayesian framework efficiently track labels, cardinality and trajectories of multiple objects**
- **MCMC implementation of the proposed tracking algorithm verifies its simplicity and accuracy.**

Multiple object tracking methods

Bayesian estimation of object state posterior using physics-based models (e.g., constant velocity, maneuvering) & parametric models (e.g., multi-Bernoulli)

- **Joint probabilistic data association**: all measurement-to-target associations **[Y. Bar-Shalom 1983]**
- **Multiple hypothesis testing**: measurement-to-track associations as multiple hypothesis **[Y. Bar-Shalom 1998]**
- **Joint multitarget probability density**: joint multitarget conditional density using independent and coupled partitioning **[A. Hero 2003]**
- **Probability hypothesis density** (PHD): model uncertainty using random finite set (RFS), approximate multi-target Bayes recursion by propagating state posterior **[J. Bell 2005, R. Mahler 2012]**
- **Multitarget Multi-Bernoulli** filter: propagate parameters of multi-Bernoulli RFS to approximate multi-object posterior; requires high P_D, low P_{FA} **[Vo & Vo 2012]**
- **Labeled Multi-Bernoulli**: use labeled RFS to estimate target identity (assuming finite number of targets) **[Vo & Vo 2014]**

Bayesian nonparametric modeling and tracking

Nonparametric prior on object states

- **Hierarchical Dirichlet process (HDP):** use as prior on unknown number of modes **[E. Fox 2009]**
- **Bayesian inference:** use Dirichlet process mixtures to model noise in linear dynamic system **[F. Caron 2008]**
- **Dependent Dirichlet process:** estimate object time-varying cardinality, state and label **[B. Moraffah 2018]**
- **Random infinite tree:** estimate time-varying cardinality with infinite random tree **[B. Moraffah 2019]**

Construction of multiple prior distribution

- **Cluster label assignments with unknown cluster parameter θ_k**
- **Three possible scenarios for an object staying in scene at time step k**

Scenario 1: object placed in survived/transitioned cluster from time ($k-1$) occupied by other clustered objects at time k

Scenario 2: object placed in survived/transitioned cluster from time ($k-1$) not occupied by other clustered object at time k

Scenario 3: object not placed in existing cluster from time ($k-1$); generate new cluster parameter at time k