METRIC-Bayes: Measurements Estimation for Tracking in High Clutter using Bayesian Nonparametrics

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https://bmoraffa.github.io/presentations
Challenges of Tracking in High Clutter

• Sensor measurements, more often than not, contain detections from false targets.

• Time-dependent number of measurements that include clutter and true sensor observations with unknown origin.

• True measurements from the target are present with some probability of detection.

• Number and location of clutter measurements are random.

• Hence, for accurate tracking, tracking algorithms must only incorporate object generated measurements.
Problem Statement

• Tracking a target in high clutter:
  • State transition equation (first Markovity is assumed)
    \[ x_k = f(x_{k-1}) + n_k \]
  • Emission equation:
    \[
    z_{n,k}^T = h(x_k) + v_{n,k} \quad \text{Target originated measurements}
    
    z_{n,k}^C \quad \text{Clutter measurements}
    \]

Where \( n = 1, 2, \ldots, N_k \), time-dependent!

• If known \( \{z_{n,k}\}_{n=1}^{N_k} \), then posterior distribution gives the target state:

\[
    p(x_k | \{z_{n,k}\}_{n=1}^{N_k})
\]

But we do NOT have the true measurements and presence of clutter deteriorates the performance!
Object Tracking in Clutter: Literature

• Strongest-neighbor and nearest-neighbor filters (NN filter)
  • Measurement that is statistically closest to the predicted measurement is from the object and the rest are clutter!
  • Object motion is linear Gaussian.
  • **Disadvantage1**: Performance diminishes as probability of false alarm rate increases.
  • **Disadvantage2**: Uses the philosophy of “winner takes it all”

• Probabilistic data association filter (PDA filter)
  • Used to validate multiple measurements according to their probability of target origin.
  • Assumes object motion obeys linear Gaussian statistics.
  • All non-object originated measurements are assumed to be clutter that is uniformly distributed in the space and Poisson distributed in time.
  • Several variations of the PDA methods proposed:
    • filtered gate structure method; interactive-multiple model PDA; Viterbi and fuzzy data association
  • **Disadvantage**: PDA type methods can become computationally intensive as the number of measurements increases.

Bayesian Nonparametric Modeling to Rescue!

• Bayesian Statistics:
  • Probabilistic modeling to express all forms of uncertainty and noise
  • … then *inverse probability* rule (i.e. Bayes’ Theorem) allows us to infer unknown quantities, learn from data, and make predictions
    • Bayes’ theorem:

\[
Q(d\theta | X = x) = \frac{dP(X \in \cdot | \theta)}{dP(x \in \cdot)} Q(d\theta)
\]

• Bayesian statistics that is not parametric (wait!)

• Bayesian nonparametrics (i.e. not finite parameter, unbounded/growing/infinite number of parameters)
  • BNP models do not generally satisfy Bayes’ theorem since the density cannot exist for all *x* (undominated models) (not the same as posterior tractability!)
  • Random discrete measures are often undominated.

• Why Bayesian nonparametrics?
  • Bayesian : Simplicity (of the framework)
  • Nonparametric : Complexity (of the real world phenomena)
Bayesian Nonparametrics in Tracking:

- Dependent Dirichlet prior, random infinite tree: time-varying cardinality and evolving states [Moraffah 2018, 2019]
- Hierarchical Dirichlet process prior: prior on unknown number of modes [Fox 2009]
- Bayesian inference: Dirichlet process mixtures for noise in dynamic system [Caron 2008]
- Graphical models for visual object recognition and tracking [Sudderth 2006]
- Learning hierarchical models of scenes, objects, and parts [Sudderth 2005]
Proposed Approach: METRIC Bayes

• Bayesian nonparametric modeling to estimate the measurements that are originated from the target

• METRIC Bayes Intuition:

  Model the marginal distributions of the joint prior as two conditionally independent Dirichlet process

• Consider a "joint" Dirichlet process prior over the parameters of true measurements and clutter

\[
p(z^T, z^C) = p(z^T | z^C)p(z^C)
\]

• Draw parameters associated with each distribution from a Dirichlet process!

[Moraffah 2020]
METRIC Bayes: Prior Distributions

- The hierarchical model describing METRIC Bayes is
  - Prior distributions on clutter parameters at time k:
    \[ G^C_k \sim \text{DP}(\alpha_C, H_C) \]
    \[ \theta_{n,k} | G^C_k \sim G^C_k \]
  
  Define: \[ \Theta_k := \{\theta_{1,k}, \ldots, \theta_{N_k,k}\} \]
  - Complete conditional prior on the parameters of true measurements at time k:
    \[ G^T_k | \Theta_k \sim \text{DP}\left(\alpha_T, H_T + \sum_{n=1}^{N_k} \delta_{\theta_{n,k}}\right) \]
    \[ \eta_{n,k} | G^T_k \sim G^T_k \]
  
  - Likelihood distributions:
    \[ z^T_{n,k} | \eta_{n,k}, \{\theta_{n,k}\} \sim F_T(\cdot | \{\eta_{n,k}\}) \]
    \[ z^C_{n,k} | \eta_{n,k}, \{\theta_{n,k}\} \sim F_C(\cdot | \{\theta_{n,k}\}) \]
METRIC Bayes: Prior Distributions

• Incorporate true measurements into the Bayesian tracker as follows:
  • Form the likelihood ratio test:

\[
\mathcal{L}(z_k^T; \{\eta_{n,k}\}, x_k) = \frac{\prod_{m} p(z_{m,k}^T|x_k; \text{target present})}{\prod_{m} p(z_{m,k}^C|x_k; \text{target absent})}
\]

• Bayesian tracker:

\[
p(x_k|z_k) \propto p(z_k^T|\{\eta_{n,k}\}, x_k) p(x_k|z_k^{T_{k-1}})
\]

• Sampling using sequential Monte Carlo or Gibbs sampling

(Details in the paper)
METRIC Bayes in one glance

Algorithm 1 METRIC-Bayes: Tracking in high clutter using jointly DPs prior.

Initialize target state: $\mathbf{x}_0$

for $k = 1 : K$ do

Predict $p\left(\mathbf{x}_k \mid \mathbf{Z}_{k-1}^{(t)}\right)$ using $p\left(\mathbf{x}_k \mid \mathbf{Z}_{k-1}^{(t)}\right) = \int p(\mathbf{x}_k \mid \mathbf{x}_{k-1}) p\left(\mathbf{x}_{k-1} \mid \mathbf{Z}_{k-1}^{(t)}\right) d\mathbf{x}_{k-1}$

Input measurements $\{\mathbf{z}_{1,k}, \ldots, \mathbf{z}_{N_{k,k}}\}$

Draw clutter prior $G_{k}^{(c)}$ from DP($\alpha_{c}, H_{c}$)

Use clutter prior to draw $\Theta_{k} \sim G_{k}^{(c)}$ iid

Draw target prior $G_{k}^{(t)}$ from $G_{k}^{(t)} \mid \Theta_{k} \sim$ DP($\alpha_{t}, H_{t} + \sum_{i=1}^{N_{k}} \delta_{\mathbf{z}_{i,k}}$)

Use target prior to draw $W_{k} \mid G_{k}^{(t)}, \Theta_{k} \sim$ $G_{k}^{(t)}$ iid

For target measurements, draw $\mathbf{z}_{n,k}^{(t)}$, using Equation (3)

For clutter measurements, draw $\mathbf{z}_{n,k}^{(c)}$, using Equation (4)

Cluster into target measurements $\mathbf{Z}_{k}^{(t)}$ with cardinality $M_{k}^{(t)}$

Cluster into clutter measurements $\mathbf{Z}_{k}^{(c)}$ with cardinality $M_{k}^{(c)}$

Compute likelihood ratio $L\left(\mathbf{Z}_{k}^{(t)} ; W_{k}, \mathbf{x}_{k}\right)$ using Equation (5)

Compute and return the posterior density using $p\left(\mathbf{Z}_{k}^{(t)} \mid \mathbf{x}_{k}, W_{k}\right)$

Update $p\left(\mathbf{x}_{k} \mid \mathbf{Z}_{k}^{(t)}\right)$ using $p\left(\mathbf{x}_{k} \mid \mathbf{Z}_{k}^{(t)}\right) \propto p\left(\mathbf{Z}_{k}^{(t)} \mid W_{k}, \mathbf{x}_{k}\right) p\left(\mathbf{x}_{k} \mid \mathbf{Z}_{k-1}^{(t)}\right)$
Experiment I: METRIC Bayes vs Bayesian Filtering

Object location estimation mean-squared error (MSE) obtained using METRIC-Bayes vs Bayesian filter that uses all the measurements

SCR = 5.9379
Experiment II: METRIC Bayes vs NN and PDF Filters

Object location estimation mean-squared error (MSE) obtained using METRIC-Bayes vs NN and PDA filters for tracking a single object.
Conclusions

• Tracking a target in clutter with unknown number of clutters.
• A class of nonparametric models based on a nested joint Dirichlet process
• No assumptions needed for prior knowledge of marginal PDFs.
• Incorporate Bayesian tracker into the modeling.
• Low computational cost as no optimization necessary
• No parametric assumption is made.
• This model can be easily generalized to track multiple objects by incorporating it into a multiple object tracking technique e.g., DDP prior on the states.